PERGAMON Mathematical and Computer Modelling 35 (2002) 1261–1269

MATHEMATICAL AND COMPUTER MODELLING

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Control of Arm Movement Using Population of Neurons

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(Received and accepted August 2001)

Abstract—Movements of the human arm in a horizontal plane are very stereotyped in the sense that the corresponding paths are mainly straight lines and the velocity profiles are "bell-shaped like" functions. A dynamics of two link model of the human arm has been studied with the goal of synthesizing the torques which accomplish the desired transfer. The parameters of the desired trajectory, as well as the system variables, are encoded using populations of a different number of neurons. The driving torques are generated from the corresponding activities using an optimal decoding rule. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords—Path plan, Internal dynamics, Neural activities, Optimal decoding.

1. INTRODUCTION

It has been experimentally verified that humans tend to reach from one point in a horizontal plane to another in a stereotyped fashion, that is, the path of a human wrist is primarily a straight line, while the corresponding velocity profile is a bell-shaped function. Moreover, the peak velocity and the distance traveled by a wrist are not independent, i.e., the longer the distance, the higher the peak velocity, which implies that the total time of transfer remains fairly constant over different experimental trials, see [1].

2. MATHEMATICAL MODEL

The model of a human arm is represented as a two-link rigid body (no muscles are assumed at this point), see Figure 1. Using standard tools from analytical mechanics, the nonlinear model of the two link system is obtained,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}, \qquad \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = -T^{-1}(x)C(x)\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + T^{-1}(x)\begin{bmatrix} \tau_s \\ \tau_e \end{bmatrix}, \tag{1}$$

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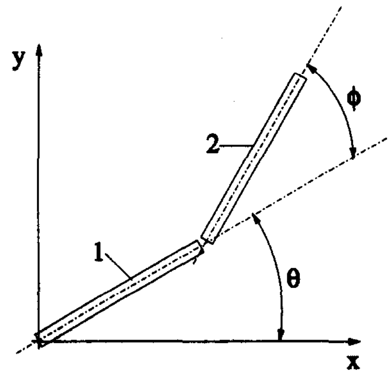


Figure 1. Two link system.

where $x = [\theta, \phi, \dot{\theta}, \dot{\phi}]^{\mathsf{T}}$, $T(x)$ represents the inertial matrix, and $C(x)$ is the matrix of Coriolis and centripetal terms.

Since the problem is constrained to a horizontal plane, there is no gravity force (1). The friction forces have been neglected as well. The torques, τ_s (shoulder) and τ_e (elbow), both acting in a horizontal plane are to be synthesized by a population of neurons. The torques are first found analytically using *feedback linearization*, a procedure for stabilizing certain class of nonlinear systems [2]. This provides an elegant solution, i.e., the synthesized torques depend both on the desired parameters and the actual position and velocity, and thus, can be viewed as a combination of feedforward/feedback signal. Without going into details of the nonlinear control theory, we give a final result for the torque pair which will cause system (1) to follow the desired trajectory parameterized by (θ_d, ϕ_d) and the desired velocity profile parameterized by $(\dot{\theta}_d, \dot{\phi}_d)$ is given by

$$\begin{bmatrix} \tau_s \\ \tau_e \end{bmatrix} = C \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} + T \begin{bmatrix} \ddot{\theta}_d + f_{12}(\dot{\theta}_d - \dot{\theta}) + f_{11}(\theta_d - \theta) \\ \ddot{\phi}_d + f_{22}(\dot{\phi}_d - \dot{\phi}) + f_{21}(\phi_d - \phi) \end{bmatrix}.$$

A similar result can be obtained using the so-called *inverse dynamics* approach, as the synthesized torques yield a cancellation in the dynamics of an arm, see [3]. The block diagram of the system has been shown in Figure 2.

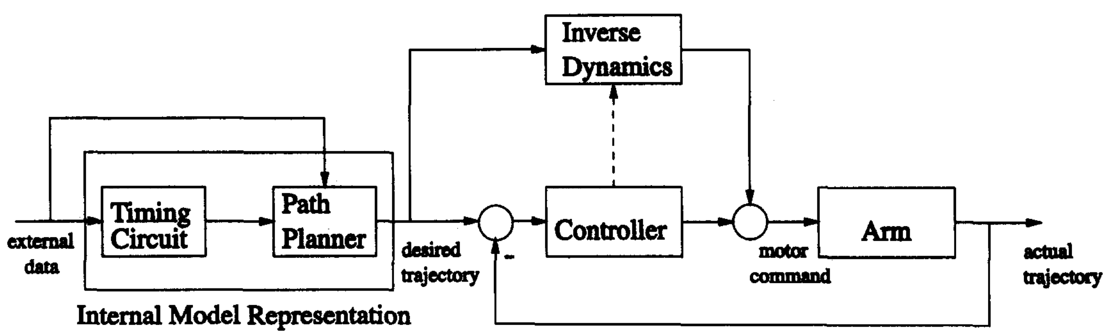


Figure 2. Block diagram of an arm control system.

3. NEURAL DYNAMICS

The point of departure in synthesizing the controlling torques using a population of neurons is a *path plan*; straight line path and bell-shaped velocity profile with known parameters such as the distance, direction, and peak velocity of the movement (Figure 3) constrain the desired angular velocities.

The initial position of the arm has been randomized by choosing the initial angles from a uniform distribution. The final position is also random in the sense that it is determined by a

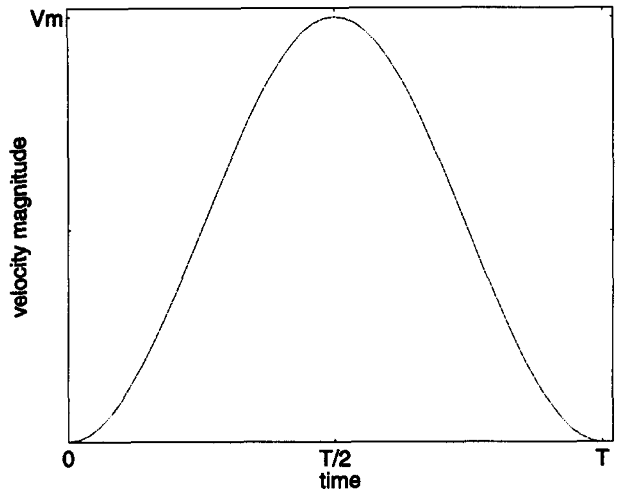


Figure 3. The velocity profile plot; V_m—peak velocity, T—total time of transfer.

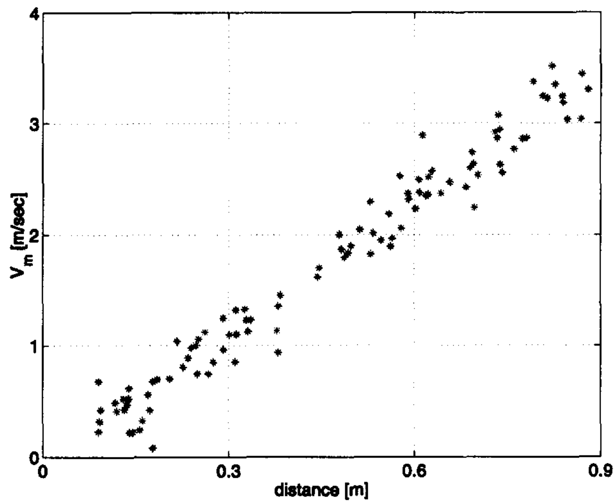


Figure 4. Correlation between the peak velocity V_m and the distance.

human decision (where we want to reach). The peak velocity is chosen randomly but with high correlation to the total distance, which is known once the initial and final position have been specified (Figure 4). This assumption has a biological relevance. If we want to reach farther, we do it with higher velocity so that the total reaching time remains constant over different trials. The average time of the transfer is assumed to be 0.5 sec. Using obvious kinematic relationships, the velocity of the wrist can be expressed as

$$\vec{V}(t) = \frac{V_m}{2}\left(1 - \cos\left(\frac{2\pi t}{T}\right)\right)\frac{\vec{r_f} - \vec{r_i}}{\delta}, \tag{2}$$

where $\vec{r_i}$ and $\vec{r_f}$ are the vectors of the initial and final positions of the arm, $\delta = \| \vec{r_f} - \vec{r_i} \|$, and $V_m/2(1 - \cos(2\pi t/T))$ is a mathematical fit of the bell-shaped function described earlier, which can be thought of as a solution to the second-order differential equation

$$\ddot{u} + \omega^2 u = \omega^2, \tag{3}$$

with zero initial conditions, where $\omega = 2\pi t/T$. Equation (3) is solved using a population of neurons, where the neurons are assumed to be responsible for encoding analog variables/vectors

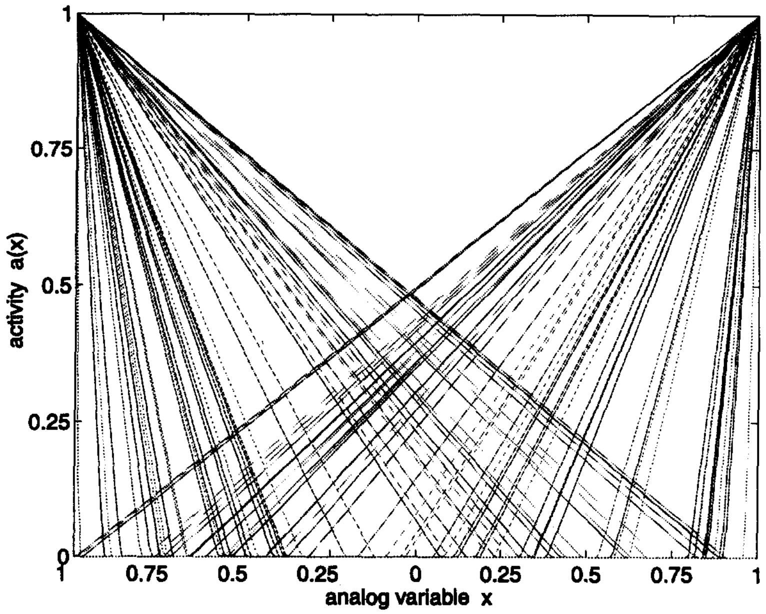


Figure 5. Normalized activity as a function of an analog variable in the range $[-1, 1]$.

using their activities. The neuronal activity is a frequency in its nature and represents the instantaneous firing rate associated with the neurons. The firing rates of the neurons are assumed to be piecewise linear, positive semidefinite functions of analog *meta* variables [4]. In the case of scalar variables taking both positive and negative values, the concept of so-called on/off cell has been used (Figure 5). For vectors, a preferred direction is what determines the extent of population response.

Therefore, the analog variable u is being encoded by a population of neurons using a nonlinear transformation $u \to a(u)$, where the shape of activity function depends on a choice of the model of firing neuron, and for integrate and fire neurons is a piecewise linear function

$$a_i(u) = [\alpha_i u + \beta_i]_+ ,$$

where $[\,]_+$ stands for a rectification symbol, as the negative activities are not physically meaningful. On the other hand, the analog variable u can be reconstructed from the activities by using a *linear decoding rule*, i.e.,

$$u^{est}(u) = \sum_{i=1}^{N} X_i \left[a_i(u) + \eta_i\right],$$

where X_i represent *optimal decoding weights* [5] in the sense that they minimize the error defined as

$$\text{Error} = \left\langle \int_{u_{\min}}^{u_{\max}} \left[u - u^{est}(u)\right]^2 du \right\rangle_\eta .$$

The symbol $\langle\,\rangle_\eta$ stands for an average over noise [6], η_i are independent identically distributed random variables with zero mean and variance σ^2, and N is the number of neurons within a population. Likewise, $\dot{u}$ is represented by the corresponding encoding and decoding rule

$$b_j(\dot{u}) = [\gamma_j \dot{u} + \delta_j]_+ , \qquad \dot{u}^{est}(\dot{u}) = \sum_{j=1}^{M} Y_j \left[b_j(\dot{u}) + \mu_j\right].$$

The differential equation (3) can now be translated at the level of activities

$$\frac{da_n(t)}{dt} = -\frac{1}{\tau}\left\{a_n(t) - \left[\sum_{i=1}^{N}\omega_{ni}^{(\text{int})}a_i(t) + \tau\sum_{j=1}^{M}\omega_{nj}^{(\text{ext})}b_j(t) + \beta_n\right]_+\right\},$$

$$\frac{db_m(t)}{dt} = -\frac{1}{\tau}\left\{b_m(t) - \left[\sum_{j=1}^{M}\omega_{mj}^{(\text{int})}b_j(t) + \tau\left(\omega^2 - \omega^2\sum_{i=1}^{N}\omega_{mi}^{(\text{ext})}a_i(t)\right) + \delta_m\right]_+\right\},$$

where the coupling weights for the oscillator (3) are given by

$$\omega_{ni}^{(\text{int})} = \alpha_n X_i, \qquad \omega_{nj}^{(\text{ext})} = \alpha_n Y_j, \qquad \omega_{mj}^{(\text{ext}int)} = \gamma_m Y_j, \qquad \omega_{mi}^{(\text{ext})} = \gamma_m X_i.$$

The solution u and $\dot{u}$ and the corresponding activities $a(t)$ and $b(t)$ are given in Figures 6 and 7, respectively. Once the timing circuit function has been generated, it is used as a driving signal for the path planner which indeed provides the desired angles and their first and second derivatives. Using (2), as well as basic kinematics of the system, the path planner is described by the nonlinear differential equation

$$\begin{bmatrix}\dot{\theta}_d \\ \dot{\phi}_d\end{bmatrix} = M^{-1}(\theta_d, \phi_d)\,\frac{V_m}{2}\left(1 - \cos\left(\frac{2\pi t}{T}\right)\right)\frac{\vec{r_f} - \vec{r_i}}{\delta}, \tag{4}$$

where $M(\theta_d, \phi_d)$ is a transformation matrix from Cartesian to polar coordinates. Equation (4) is to be solved by another population of neurons, where $\vec{r_i}$ and $\vec{r_f}$ are encoded using a rule of *preferred direction*. This can be thought of as *internal dynamics* [7], and represents a feedforward path in the synthesis of the control torques (Figure 2). Likewise, the actual position and velocities can be encoded using another populations of neurons, which comes from the human sensory system and are included in the structure of the torques as a feedback path. The neuronal activities are often referred to as *explicit space*, because biologists can make direct measurements on them, see [4]. Finally, the two torques are encoded by ensembles of neurons, whose activities (firing rates) involve the whole set (vector) of analog variables, e.g., the desired and actual angles,

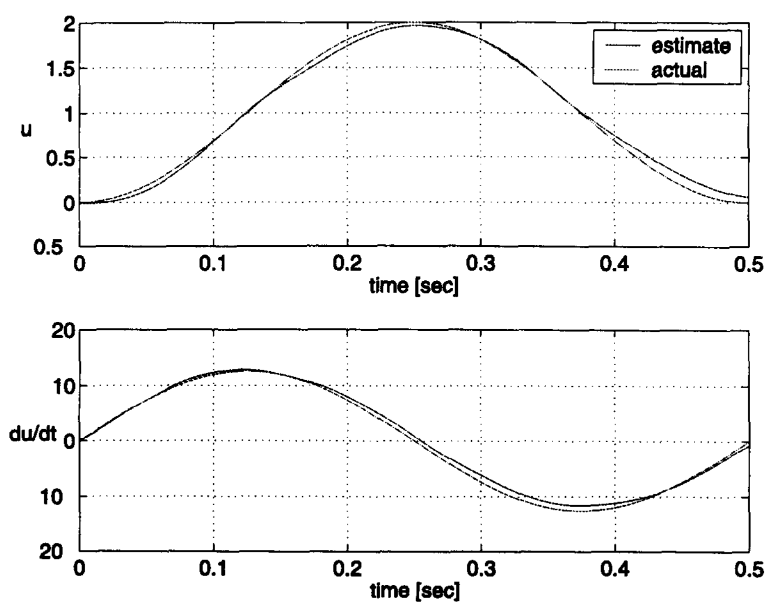
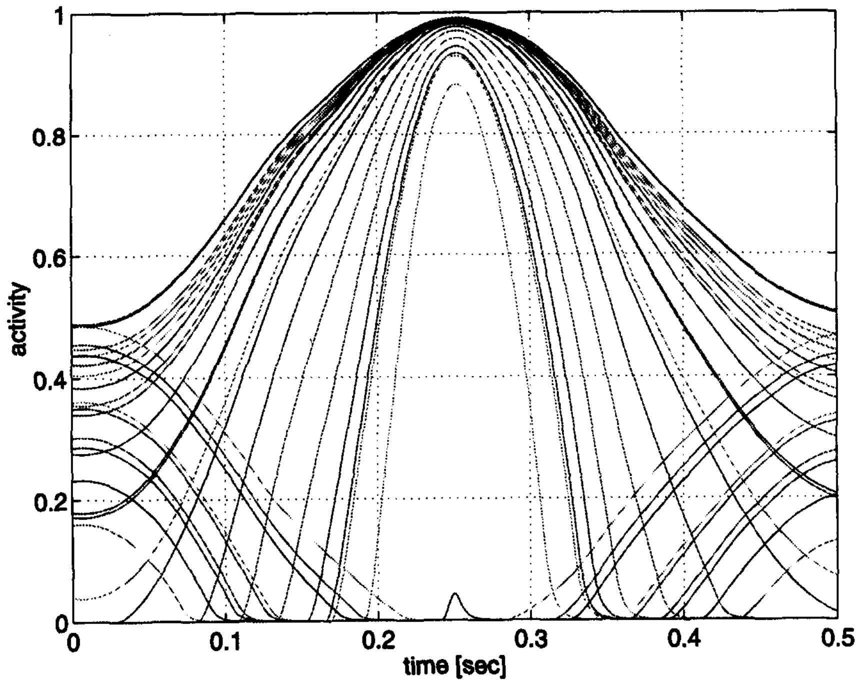
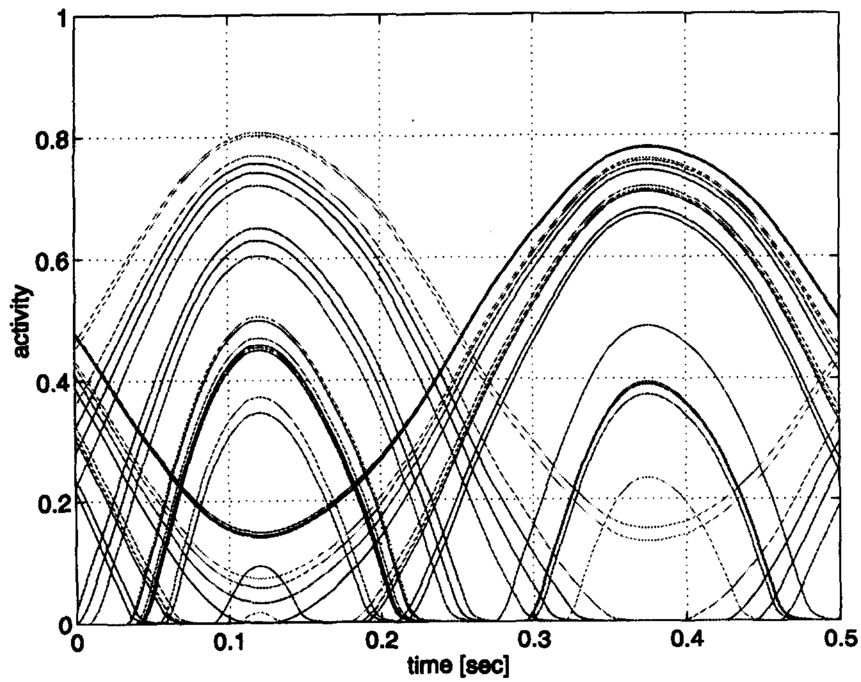


Figure 6. The solution of the oscillator (timing circuit) given by (3).



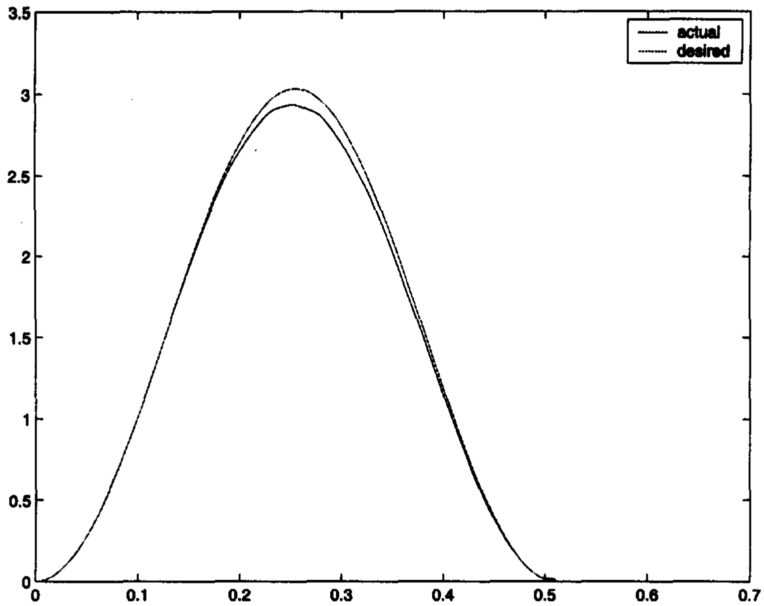
(a) $a(t)$.



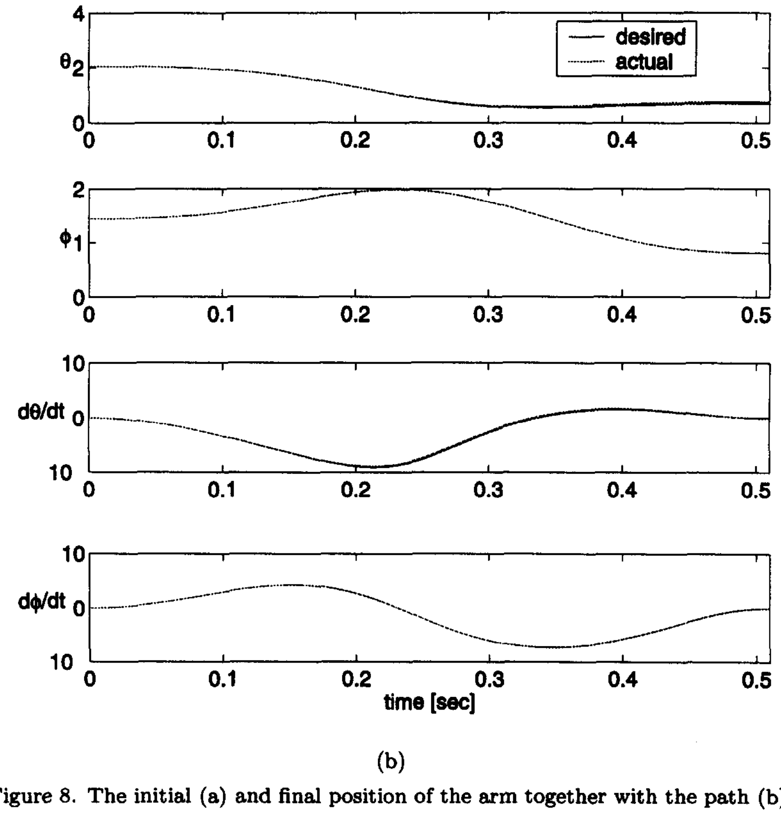
(b) $b(t)$.

Figure 7. The neural activities as functions of time.

desired and actual velocities, etc. The torques are then obtained from the activities using optimal decoding rule, in the same way it was described earlier. The procedure described above has been tested by simulations, and the deviations of the actual path from the desired one (straight line) are negligible (Figure 8). Also, the specified velocity profile and angles have been followed quite accurately (Figure 9) which is not surprising as a population of 100–150 neurons does fairly well in terms of the precision of representation.



(a)



(b)

Figure 8. The initial (a) and final position of the arm together with the path (b).

4. CONCLUSION

The experimental evidence shows that the movements of the human arm in a horizontal plane are very simple. Very often in a task of reaching from one point to another, we use a straight line path and a bell shaped velocity profile. The question of interest is how do we generate the torques which will make the arm accomplish the desired transfer while following the constraints imposed on it? In robotics, tracking the desired trajectory is a very standard problem, which can easily be solved using feedback linearization. This procedure is elegant in the sense that the

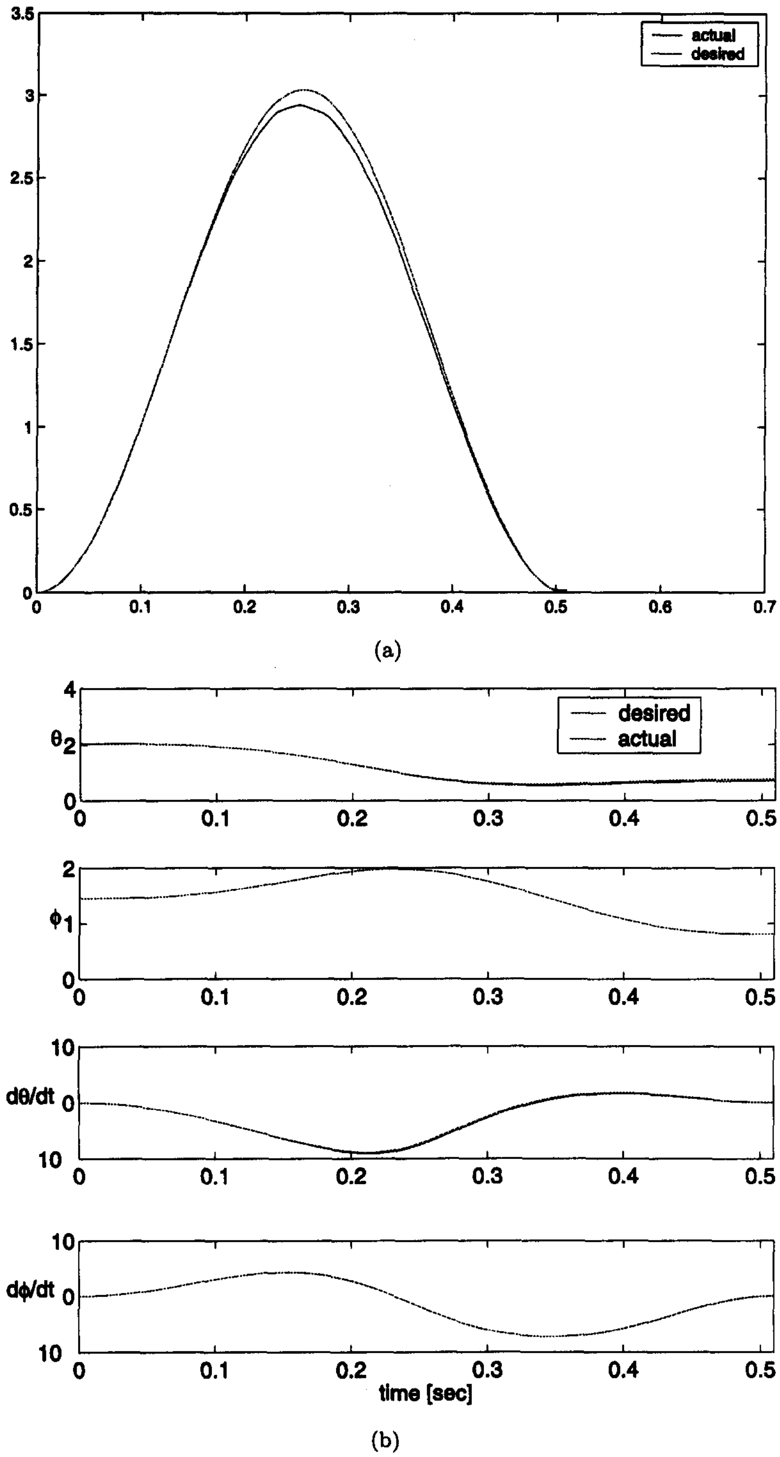


Figure 9. The desired and actual velocity profiles (a) and the desired and actual angles and angular velocities (b).

generated control (torques) are synthesized through feedforward/feedback subsystems, where the feedforward action can be attributed to a specific neural circuitry such as the cerebellum. The future work would be in the direction of making a more realistic model of the human arm, using muscles and their dynamics. The activities will then be driving the muscles, which will respond by exerting forces and torques, consequently. Also, the other types of movements can be studied, not necessarily in a horizontal plane, which certainly is a harder and more challenging problem.

REFERENCES

1. A.J. Bastian, T.A. Martin, J.G. Keating and W.T. Thach, Cerebellar ataxia: Abnormal control of interaction torques across multiple joints, *J. Neurophysiol.* **76**, 492–509 (1996).
2. A. Isidori, *Nonlinear Control Systems*, Springer-Verlag, (1989).
3. N. Schweighofer, J. Spoelstra, M.A. Arbib and M. Kawato, Role of the cerebellum in reaching quickly and accurately: II A detailed model of the intermediate cerebellum, *European J. Neurosci.* (1997).
4. C. Eliasmith and C.H. Anderson, Developing and applying a toolkit from a general neurocomputational framework, *Neurocomputing* **26/27**, 1013–1018 (1999).
5. C. Eliasmith and C.H. Anderson, Rethinking central pattern generators: A general approach, In *Proc. 8^{th} Annual Computational Neuroscience*, pp. 1013–1018, (1999).
6. F. Rieke, D. Warland, R. Steveninck and W. Bialek, *Spikes—Exploring the Neural Code*, MIT Press, (1997).
7. E. Nakano, H. Imamizu, R. Osu, Y. Uno, H. Gomi, T. Yoshioka and M. Kawato, Quantitative examinations of internal representations for arm trajectory planning: Minimum commanded torque change model, *J. Neurophysiol.* **81**, 2140–2155 (1999).